Midterm Exam — Complex Analysis

Aletta Jacobshal 03, Monday 15 December 2014, 09:00 - 11:00

Duration: 2 hours

Instructions

1. The test consists of 5 questions; answer all of them.
2. The number of points for each question is indicated at the beginning of the question. 10 points are "free" and the total number of points is divided by 10. The final grade will be between 1 and 10.

Question 1 (20 points)

Consider the function

$$f(z) = (x^2 + y^2) + 2ix,$$

where $z = x + iy$.

(a) Find the points (if any) in $\mathbb{C}$ where f is differentiable.

(b) Explain if f is analytic at the points you found in subquestion (a).

Question 2 (20 points)

It is given that the prinicipal value of the arc tangent of a complex number is

$$\operatorname{Tan}^{-1} z = \frac{i}{2} \operatorname{Log} \frac{1 - iz}{1 + iz}.$$

(a) Determine the principal value of the arc tangent of $2 + i$.

(b) For which complex numbers z is the function $\operatorname{Tan}^{-1} z$ discontinuous (or not defined) at z?

Question 3 (15 points)

Compute the integral

$$\int_\Gamma z^2 \, dz,$$

where Γ is any contour in $\mathbb{C}$ that starts at 1 and ends at $-1 + i$.

Question 4 (20 points)

Compute the value of the integral

$$\int_{\Gamma} \frac{2e^z}{(z-1)^2(z-3)}\, dz$$

where Γ is the closed contour shown in Figure 1.

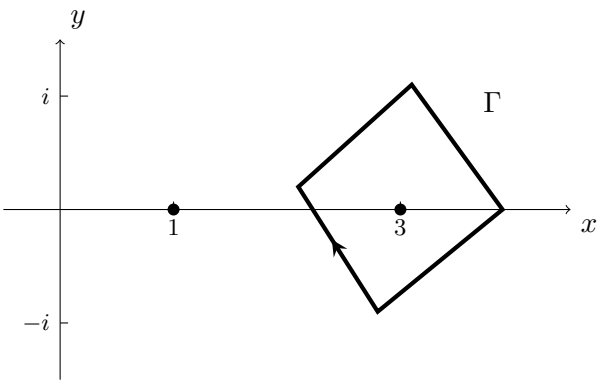

Figure 1: Contour Γ for Question 4.

Question 5 (15 points)

Consider a function $f(z)$ analytic in a domain D. Prove that if $|f(z)|^2$ is constant in D then the function $f(z)$ is constant in D.

End of the test (90 points)